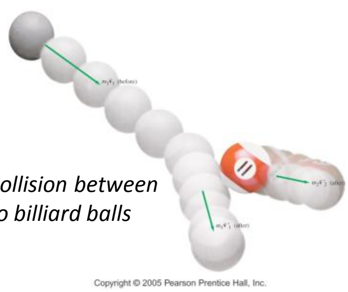
PHYS131 Mechanics
Lecture 18: Linear Momentum



A collision between two billiard balls

- Linear momentum
- Linear momentum in relation to force
- Total momentum
- Conservation of momentum
- Collisions
- Impulse
- Conservation of energy in collisions
- Collisions in 1d

Linear Momentum

- Linear momentum is the quantity mass times velocity:

$$\vec{p} = m\vec{v}$$

- It is called **linear** because it is associated with translational motion, whereas angular momentum associated with rotation
  - Often, we will just say "momentum" instead of "linear momentum"
- Momentum is a vector
- Momentum has unit: kg m s^{-1}
- Momentum is a useful quantity in physics because the momentum of a system of objects is conserved if no external force acts on the system

Momentum in relation to Force

- The rate of change of momentum of an object is equal to the net force applied to that object:

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t} = m\vec{a} = \vec{F}_{net}$$

- Hence Newton's Second Law may be written as:

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

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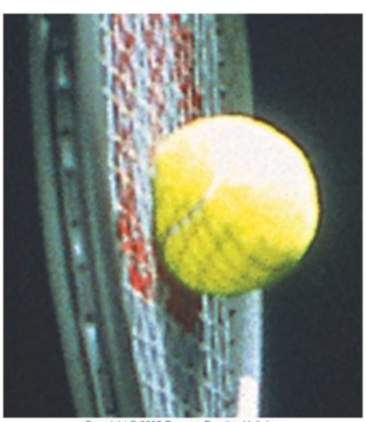
Collisions and Impulse



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During a collision, a large force acts for a short time Δt.

Since $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$, we can relate change of momentum to the force applied:

$$\vec{F}\Delta t = \Delta\vec{p}$$

This is called the impulse $\vec{J}$, defined by:

$$\vec{J} = \vec{F}\Delta t$$

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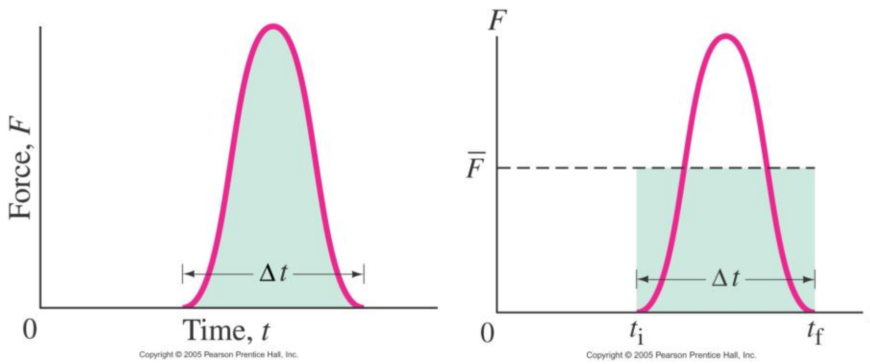
Force vs Time in a Collision

- The force varies as shown over the short time it is applied
- We do not need to know how the force varies; all we need to know is the average force during the time interval, so $J = \bar{F}\Delta t$



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Example

Q: A tennis ball of mass 60 g leaves a racket with a speed of 55 m/s. The ball is in contact with the racket for 4 ms. Find the average force exerted by the racket on the ball.

A: Impulse $J = \Delta p = m\Delta v = 0.06\text{ kg} \times (55\text{ m s}^{-1} - 0)$
$= 3.3\text{ kg m s}^{-1}$

Since $J = F_{av}\Delta t$, we have $F_{av} = \frac{J}{\Delta t} = \frac{3.3\text{ kg m s}^{-1}}{0.004\text{ s}} = 825\text{ N}$

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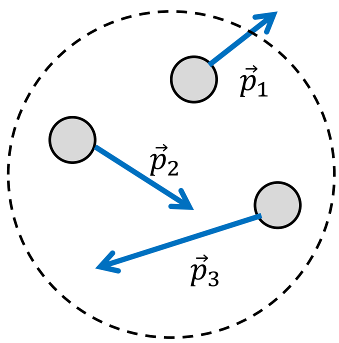
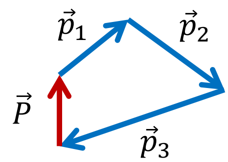
Total Momentum

- The total momentum of a system is the sum of the momentum of each object in the system



System of objects



- Total momentum is given by
$$\vec{P} = \sum_i \vec{p}_i$$
- It can also be called $\vec{p}_{total}$

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Examples

- **Q:** Calculate the momentum (magnitude) of a 4 kg mass moving at speed 12 m/s.
- **A:** *p* = *mv* = 4 x 12 = 48 kg m/s

- **Q:** In an *x*-*y* coordinate system, a system consists of 3 objects that have masses and velocities $m_1 = 2$ kg, $\vec{v}_1 = (1,3)\ \mathrm{m\ s^{-1}}$, $m_2 = 5$ kg, $\vec{v}_2 = (-2,-4)\ \mathrm{m\ s^{-1}}$, and $m_3 = 3$ kg, $\vec{v}_3 = (5,-2)\ \mathrm{m\ s^{-1}}$. Find the total momentum of the system.
- **A:** Find momenta: $\vec{p}_1 = m_1\vec{v}_1 = 2(1,3) = (2,6)\ \mathrm{kg\ m\ s^{-1}}$
$\vec{p}_2 = m_2\vec{v}_2 = 5(-2,-4) = (-10,-20)\ \mathrm{kg\ m\ s^{-1}}$
$\vec{p}_3 = m_3\vec{v}_3 = 3(5,-2) = (15,-6)\ \mathrm{kg\ m\ s^{-1}}$
- Add the three momentum vectors to get the total momentum:
$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = (2,6) + (-10,-20) + (15,-6) = (7,-20)\ \mathrm{kg\ m\ s^{-1}}$

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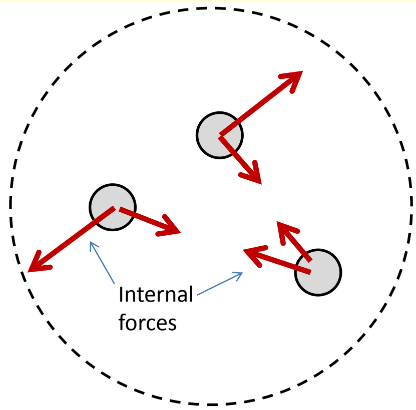
Conservation of Momentum

- Total momentum of a system is conserved (stays constant) if no external forces act on the system
- The system is **isolated**
- There can be internal forces, i.e. forces between objects in the system
- Momentum can be exchanged between objects in the system



No external forces act on objects in the system

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Collisions

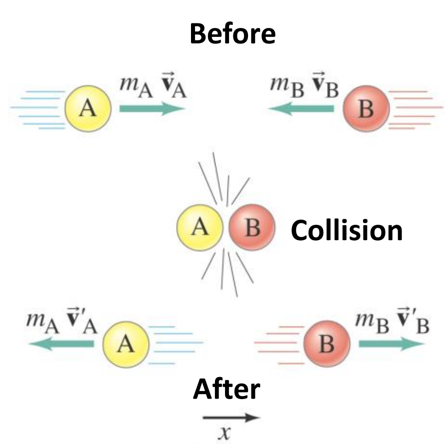
- In a collision between two objects, the total momentum is conserved if the system is isolated
- Forces act only at the instant of collision, and transfer momentum between objects
- Total momentum before the collision ($\vec{P}$) equals total momentum after the collision ($\vec{P}'$):

$$\vec{P} = \vec{P}'$$

i.e.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

- **Note** that this is a vector equation



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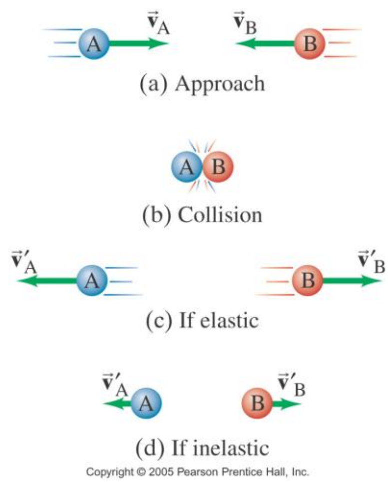
Conservation of Energy in Collisions

- Total mechanical energy (*KE* + *PE*) may or may not be conserved
- In an **elastic collision**, total mechanical energy is conserved
- In an **inelastic collision**, total mechanical energy is not conserved: some is lost



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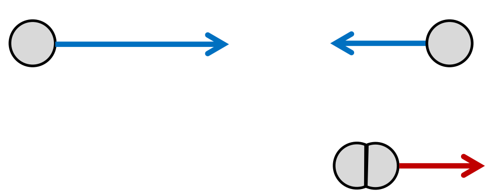
Completely Inelastic Collision

- In a **completely inelastic collision**, the objects stick together after the collision, so there is one final velocity
- Some of the total **mechanical energy** is lost, and the energy lost is not destroyed but converted to heat or internal motion or vibration of the stuck-together system

Before

After



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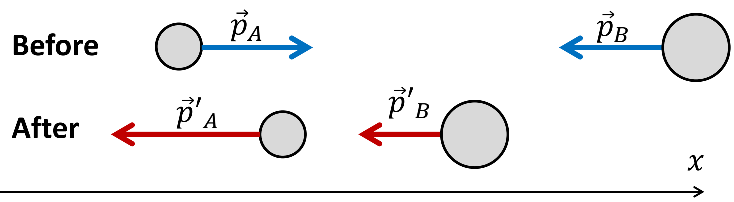
Collisions in 1d

- In a 1d collision, all momenta lie in a line



- The vector equation $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$ may be written in terms of scalar components:

$$p_A + p_B = p_A{}' + p_B{}'$$

or

$$m_A v_A + m_B v_B = m_A v_A{}' + m_B v_B{}'$$

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Collision Equations in 1d

- For a 1d collision, the equations are as follows, for velocity components in a specified reference frame
- Conservation of momentum:

$$m_A v_A + m_B v_B = m_A v_A{}' + m_B v_B{}'$$

- Conservation of mechanical energy in an elastic collision (consider only kinetic energy in this case):

$$\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2$$

- Generally, conservation of mechanical energy is described by:

$$KE_A + PE_A = KE_A' + PE_A' + E_{lost}$$

where $E_{lost} = 0$ for an elastic collision and $E_{lost} > 0$ for an inelastic collision

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Examples

Examples